2019 VI 12 1430 Seat No. | | | | | |

Time : 2½ Hours

MATHEMATICS & STATISTICS

Subject Code

H	6	0	6

Total No. of Questions : 30 **(Printed Pages : 8)** **Maximum Marks : 80**

INSTRUCTIONS :

(*i*) *All* questions are compulsory.

(*ii*) The question paper consists of **30** questions divided into five section A, B, C, D and E.

(*iii*) Section A contains **7** questions of **1** mark each, which are multiple choice type questions. Section B contain **7** questions of 2 marks each, Section C contains **7** questions of 3 marks each, Section D contains **7** questions of **4** marks each and Section E contains **2** questions of **5** marks each.

(*iv*) There is no overall choice in the paper. However internal choice is provided in **2** questions of **3** marks, **2** questions of **4** marks and **2** questions of **5** marks. In questions with choice, only one of the choices is to be attempted.

(*v*) Use of calculators is not permitted.

(*vi*) Logarithmic tables will be supplied on request.

(*vii*) Graphs should be drawn on the answer paper only.

Section A

Question Nos. **1** to **7** carry **1** mark each. In each question, *four* options are provided out of which *one* is correct. Write the correct option.

1. If A is a square matrix of order 3×3, then $(kA)^T$, $k \neq 0$ is equal to ....

- kA
- kA^T
- $\frac{1}{k}A^T$
- k^3A^T

2. A binary operation $*$ defined on Q, the set of rational numbers, as $a * b = ab + 3$, then $*$ is ....

- Associative but not commutative
- commutative but not associative
- Both commutative and associative
- Neither commutative nor associative

3. If the Banker's discount on a bill of Rs. 25,000 due in 6 months is Rs. 500 then the rate of interest is .... per annum.

- 2%
- 3%
- 4%
- 5%

4. $\int \frac{1}{\sqrt{x^2 - a^2}} dx =$ ....

- $\log|x - \sqrt{x^2 - a^2}| + c$
- $\log|x + \sqrt{x^2 - a^2}| + c$
- $\log|x + \sqrt{x^2 + a^2}| + c$
- $\log|x - \sqrt{x^2 + a^2}| + c$

5. If $A = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ then $2A - 3B =$ ....

- $\begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} -4 & -5 \\ -9 & -12 \end{bmatrix}$
- $\begin{bmatrix} 4 & 12 \\ -9 & 3 \end{bmatrix}$
- $\begin{bmatrix} -2 & 1 \\ -5 & 12 \end{bmatrix}$

6. Present value of an annuity of Rs. 600 payable at the end of each 6 months for 5 years, if money is worth 6% per annum compounded semi-annually is ....

($a_{\overline{10}/0.03} = 8.5302$, $a_{\overline{10}/0.06} = 7.3601$)

- 5223.6
- 4312.1
- 4416.06
- 5118.12

7. In a partnership deed, three partners invested Rs. 11,000, Rs. 12,000 and Rs. 13,000. then the sum of their profit sharing ratio is ....

- 16
- 26
- 36
- 52

Section B

Question Nos. **8** to **14** carry **2** marks each.

8. Using determinants, find the equation of the line joining the points (2, 3) and (4, 6).

9. Find the derivative of the function $f(x) = \log(x + 8)$. with respect to x, using the first principle.

10. A company produces two types of foods F_1 and F_2. They cost Rs. 50 and Rs. 30 per unit. Each type of food contains two types of nutrients N_1 and N_2 in different quantities. Each unit of F_1 contains 2 units of N_1 and 4 units of N_2. Each unit of F_2 contains 3 units of N_1 and 2 units of N_2. The minimum daily requirements of N_1 and N_2 is 6 units and 8 units respectively. Write the objective function and the constraints of the linear programming problem to meet the minimum requirements.

11. Define Sacrificing ratio and gaining ratio in a partnership deed.

12. Evaluate :

$$\int_1^2 \frac{2x^2 + 3x + 1}{x} dx.$$

13. A coin is tossed 3 times. If getting a head is success, find the probability of getting atleast one success.

14. Find :

$$\int \frac{5}{x(3 + 2 \log x)} dx.$$

Section C

Question Nos. **15** to **21** carry **3** marks each.

15. Siya and Riya entered into a partnership investing Rs. 42,000 and Rs. 70,000 respectively and agreed to share their profits in the ratio of their capitals. Siya is a working partner and gets Rs. 1,000 per month as working allowance. Find the total earnings of each partner in a profit of Rs. 33,700 after one year.

16. Check whether the relation R defined on the set of real numbers as $R = \{(a, b) : a \leq b\}$ is Reflexive, symmetric and transitive.

Or

If $f : R \to R$ is defined by $f(x) = 4x + 5$, show that the function $f(x)$ is one-one and onto. Hence find inverse of the function.

17. If $x^8 y^2 = (x + y)^{10}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

Or

If $y = A \cos(3x + 2) + B \sin(3x + 2)$, then prove that $\frac{d^2y}{dx^2} + 9y = 0$.

18. Evaluate :

$$\int_0^{\pi/2} \frac{\sin^2 x}{(1 + \cos x)^2} dx.$$

19. If $x = 3(t - \sin t)$, $y = 3(1 - \cos t)$ find $\frac{dy}{dx}$.

20. Solve the differential equation $x \log x \, dy - y \, dx = 0$.

21. Show that the following differential equation is homogeneous and hence solve it :

$$x \frac{dy}{dx} = x + y.$$

Section D

Question Nos. **22** to **28** carry **4** marks each.

22. By using properties of determinants show that :

$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3.$$

23. Solve the following system of equations, using matrix method :

$$x + 2y + z = 7$$

$$x + 3z = 11$$

$$2x - 3y = 1.$$

24. Define removable type of discontinuity of a function at a point. Determine whether the function $f(x)$ defined below is continuous at $x = 0$, where :

$$f(x) = \frac{\sqrt{x+1} - 1}{\log(1+x)}; \qquad x > 0$$

$$= \frac{2x^2 + 3x}{6x}; \qquad x = 0$$

$$= \frac{\sin 3x}{\tan 4x}; \qquad x < 0.$$

25. Find :

$$\int \frac{1}{3 \sin x + 4 \cos x + 5} dx.$$

Or

Find :

$$\int \frac{e^x \, dx}{(e^x - 1)(e^x + 3)(2e^x - 1)}$$

26. Solve the following linear programming problem graphically.

Maximise :

$$Z = 5x + 10y$$

Subject to the constraints :

$$x + 2y \leq 120$$

$$x + y \geq 60$$

$$x - 2y \geq 0, \qquad x, y, \geq 0$$

27. Two rotten apples are accidentally mixed with eight good ones. Two apples are drawn one after another with replacement from this lot. Find the probability distribution of the number of rotten apples.

Or

Of the students in a college, it is known that 60% reside in city and 40% reside in village. Previous year results report that 30% of all students who reside in the city attain A grade and 20% of the village students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is from city.

28. A bill of exchange for Rs. 10,000 drawn on 19 June for 5 months was discounted for Rs. 9900 at the rate of 5% p.a. Find the date on which bill was discounted.

Section E

Question Nos. **29** to **30** carry **5** marks each.

29. A machine costs a company Rs. 63,000 and its effective life is estimated to be 12 years. A sinking fund is created in order to replace the machine by a new model at the end of its lifetime. The scrap of the old machine would yield Rs. 3,000. Find what amount should be set aside at the end of each year to accumulate at compound interest of 15% per year. (Use logarithmic table).

Or

Find the present value of annuity of Rs. 600 payable at the beginning of each 3 months for a period of 15 years. If the interest rate is 6% p.a. compounded quarterly. (Use logarithmic table).

30. The demand equation for a manufacturer's product is $p = 8000 - 40x - x^2$, where p is the price per unit and x is the number of units sold. At what level of output, x, the total revenue will be maximum ? Also find the price per unit

Or

If $C = 5x^3 - 24x^2 - \frac{x^4}{3} - 20x$ is a cost function, find the average cost function. At what level of production x, is there a minimum average cost ?